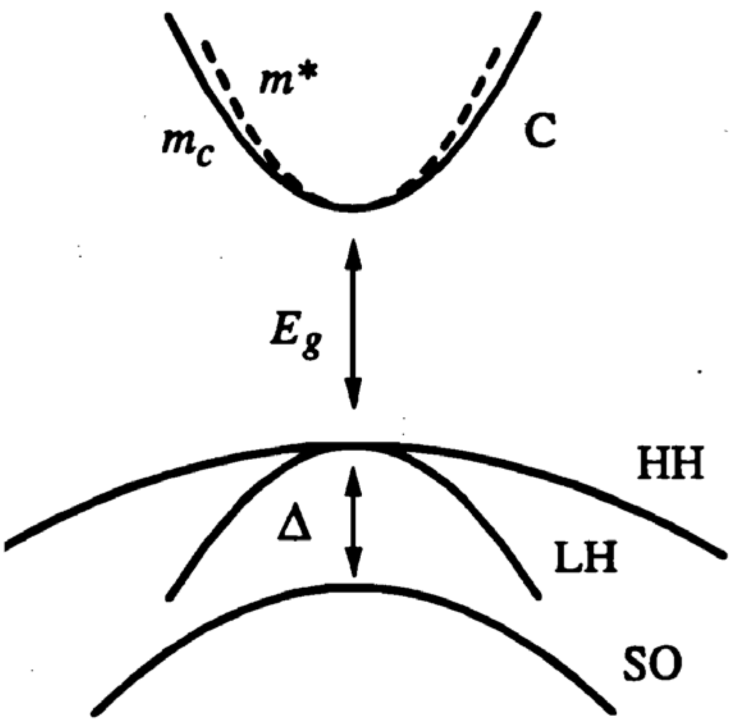
EE 232 Lightwave Devices
Lecture 15: Polarization Dependence

Reading: Chuang, Sec. 4.1-4.2, 9.5
(There is also a good discussion in Coldren, Appendix 8 and 10)

Instructor: Ming C. Wu

University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

Detailed Band Structure



Block wavefunction: $\psi_{nk}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{nk}(\vec{r})$

At band edges, $\vec{k} = 0$

Conduction Band: Remnant of s atomic orbital

$$u_{c0}(\vec{r}) = |S\rangle$$

Valence Band: remnants of the p orbitals:

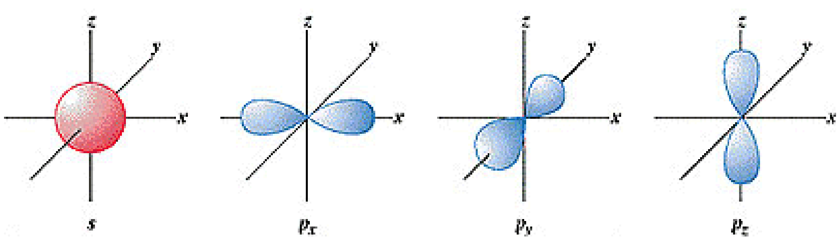
$$u_{v0}(\vec{r}) = |X\rangle, |Y\rangle, |Z\rangle$$

$|S\rangle$ is symmetric in x, y, and z

$|X\rangle$ is anti-symmetric in x, and symm in y, z

$$\langle S|X\rangle = \langle S|Y\rangle = \langle S|Z\rangle = 0$$

$$\langle S|P_x|X\rangle = \langle S|P_y|Y\rangle = \langle S|P_z|Z\rangle \neq 0$$



$|S\rangle$, $|X\rangle$, $|Y\rangle$, $|Z\rangle$

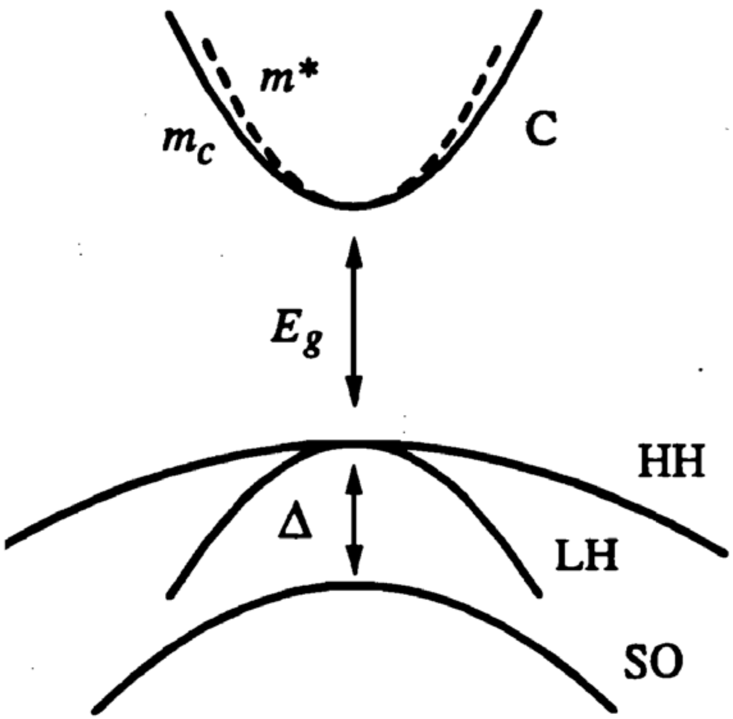
Wavefunctions for Electrons and Various Holes (heavy, light holes)

Define z-direction along $\vec{k}$ (i.e., $\vec{k} = k\hat{z}$)

CB wavefunctions:

$$u_c = \left| iS \uparrow \right\rangle$$

$$\bar{u}_c = \left| iS \downarrow \right\rangle$$



VB wavefunctions:

$$u_{hh} = \left| \frac{3}{2}, \frac{3}{2} \right\rangle = \frac{-1}{\sqrt{2}} \left| (X + iY) \uparrow \right\rangle$$

$$\bar{u}_{hh} = \left| \frac{3}{2}, \frac{-3}{2} \right\rangle = \frac{1}{\sqrt{2}} \left| (X - iY) \downarrow \right\rangle$$

$$u_{lh} = \left| \frac{3}{2}, \frac{1}{2} \right\rangle = \frac{-1}{\sqrt{6}} \left| (X + iY) \downarrow - 2Z \uparrow \right\rangle$$

$$\bar{u}_{lh} = \left| \frac{3}{2}, \frac{-1}{2} \right\rangle = \frac{1}{\sqrt{6}} \left| (X - iY) \uparrow + 2Z \downarrow \right\rangle$$

$$u_{so} = \left| \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}} \left| (X + iY) \downarrow + Z \uparrow \right\rangle$$

$$\bar{u}_{so} = \left| \frac{1}{2}, \frac{-1}{2} \right\rangle = \frac{1}{\sqrt{3}} \left| (X - iY) \uparrow - Z \downarrow \right\rangle$$

Quantum Well Matrix Element

	TE $\hat{e} = \hat{x}$ or $\hat{y}$	TM $\hat{e} = \hat{z}$	All Polarizations (2xTE + TM)
C-HH Transition	$\frac{3}{4}\left(1+\cos^2\theta\right)M_b^2$	$\frac{3}{2}\left(\sin^2\theta\right)M_b^2$	$3M_b^2$
C-LH Transition	$\left(\frac{5}{4}-\frac{3}{4}\cos^2\theta\right)M_b^2$	$\left(\frac{1}{2}+\frac{3}{2}\cos^2\theta\right)M_b^2$	$3M_b^2$
Sum Rule: HH + LH	$2M_b^2$	$2M_b^2$	$6M_b^2$

Prof. Ming Wu

Angular Factor

Angular factor : $\cos^2\theta = \frac{k_z^2}{k^2} = \frac{k_z^2}{k_t^2 + k_z^2}$

In quantum wells, k_z is quantized: $E_{en} = \frac{\hbar^2 k_z^2}{2m_e^*}$

$$\cos^2\theta = \frac{\frac{\hbar^2 k_z^2}{2m_e^*}}{\frac{\hbar^2 k_t^2}{2m_e^*} + \frac{\hbar^2 k_z^2}{2m_e^*}} = \frac{E_{en}}{\frac{\hbar^2 k_t^2}{2m_e^*} + E_{en}}$$

k_t can be estimated from $\frac{\hbar^2 k_t^2}{2m_r^*} = \hbar\omega - E_g - E_{en} - |E_{hm}|$

At band edge, $k_t = 0 \quad \Rightarrow \quad \cos^2\theta = 1$

Quantum Well Matrix Element for Light-Matter Interaction Near Bend Edges

	TE $\hat{e} = \hat{x}$ or $\hat{y}$	TM $\hat{e} = \hat{z}$	All Polarizations (2xTE + TM)
C-HH Transition	$\frac{3}{2} M_b^2$	0	$3M_b^2$
C-LH Transition	$\frac{1}{2} M_b^2$	$2M_b^2$	$3M_b^2$
Sum Rule: HH + LH	$2M_b^2$	$2M_b^2$	$6M_b^2$

Prof. Ming Wu

Transition Strength versus Transverse Wavevector

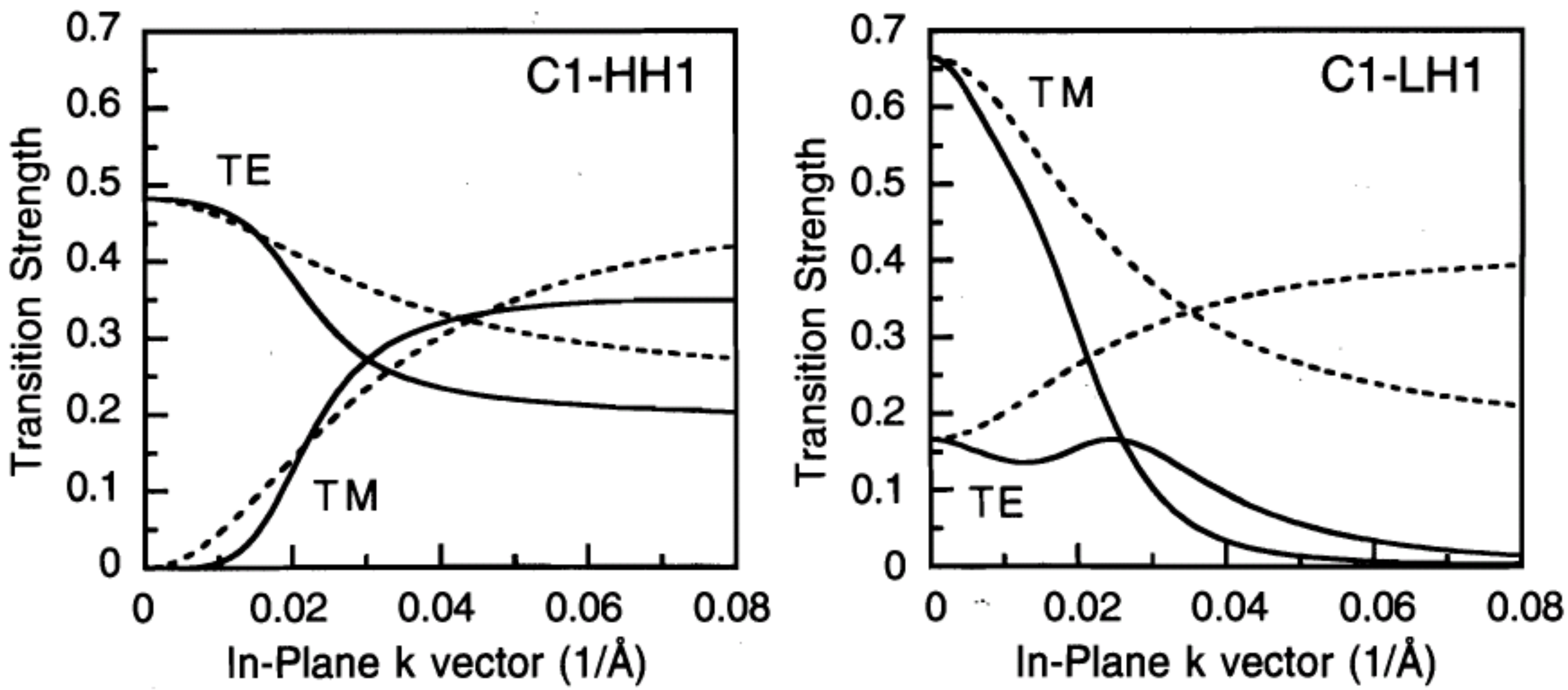


FIGURE A10.5 Relative transition strengths for both TE and TM light polarization for the two lowest subband transitions in an unstrained GaAs/$Al_{0.2}Ga_{0.8}As$ 80Å QW. The dashed curves represent what one would calculate assuming parabolic subbands. The transition strength as plotted here is defined as $|M_T|^2/|M|^2$ (bulk value is 1/3).

Appendix:
$k \cdot P$ Method and Derivation of Matrix Elements

k·P Method

$$H\psi_{nk}(\vec{r}) = [-\frac{\hbar^2}{2m_0}\nabla^2 + V]\psi_{nk}(\vec{r}) = E_n(\vec{k})\psi_{nk}(\vec{r})$$

$$\psi_{nk}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_{nk}(\vec{r})$$

$u_{nk}(\vec{r}+\vec{R}) = u_{nk}(\vec{r})$ periodic function, $\vec{R}$ is lattice vector

$$\left[\frac{P^2}{2m_0} + \frac{\hbar}{m_0}\vec{k}\cdot\vec{P} + V(\vec{r})\right]u_{nk}(\vec{r}) = \left[E_n(\vec{k}) - \frac{\hbar^2 k^2}{2m_0}\right]u_{nk}(\vec{r})$$

Near $\vec{k}=0$, $\vec{k}\cdot\vec{P}$ can be treated as a perturbation

$$\left[H_0 + \frac{\hbar}{m_0}\vec{k}\cdot\vec{P}\right]u_{nk}(\vec{r}) = \left[E_n(\vec{k}) - \frac{\hbar^2 k^2}{2m_0}\right]u_{nk}(\vec{r})$$

Kane's P parameter: $P = \frac{-i\hbar}{m_0}\langle S|P_z|Z\rangle$

Second-Order Perturbation

Conduction Band:

Second-order perturbation:

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + \frac{\hbar^2}{m_0^2} \sum_{n \neq c} \frac{\left|\vec{k} \cdot \vec{P}_{cn}\right|^2}{E_c(0) - E_n(0)}$$

$$\vec{P}_{cn} = \left\langle u_c \middle| \vec{P} \middle| u_n \right\rangle, \quad n = hh,\ lh,\ so$$

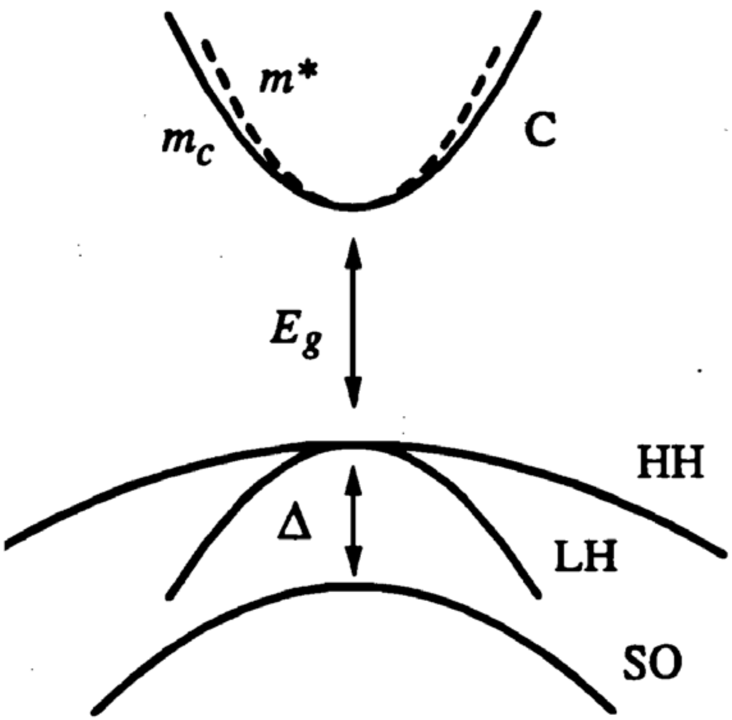
Use Kane's P parameter:

$$P = \frac{-i\hbar}{m_0} \left\langle S \middle| P_z \middle| Z \right\rangle$$

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + k^2 P^2 \left(\frac{2}{3} \frac{1}{E_g} + \frac{1}{3} \frac{1}{E_g + \Delta} \right)$$

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + \frac{k^2 P^2}{3} \left(\frac{3E_g + 2\Delta}{E_g (E_g + \Delta)} \right) = \frac{\hbar^2 k^2}{2m_e^*}$$

Eigenvalues



Conduction Band:

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + \frac{k^2 P^2}{3}\left(\frac{3E_g + 2\Delta}{E_g(E_g+\Delta)}\right) = \frac{\hbar^2 k^2}{2m_e^*}$$

Valence Band:

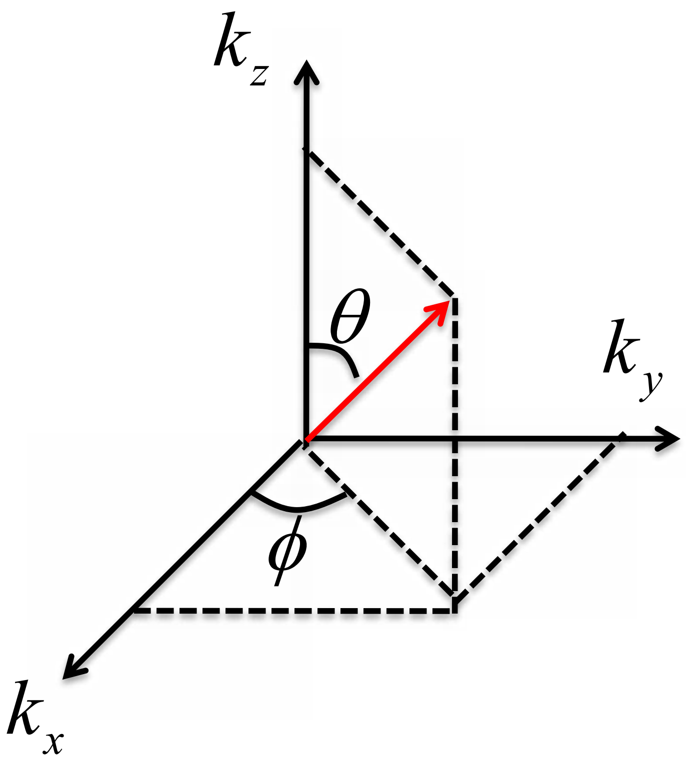
HH: $E_{hh}(\vec{k}) = \frac{\hbar^2 k^2}{2m_0}$ (incorrect in this approx)

LH: $E_{lh}(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} - \frac{2k^2 P^2}{3E_g} = -\frac{\hbar^2 k^2}{2m_{lh}^*}$

SO: $E_{so}(\vec{k}) = -\Delta + \frac{\hbar^2 k^2}{2m_0} - \frac{2k^2 P^2}{3\left(E_g + \Delta\right)} = -\Delta + -\frac{\hbar^2 k^2}{2m_{so}^*}$

Wavefunctions in General Coordinates

When $\vec{k}$ is not along z direction (i.e., $\vec{k} \neq k\hat{z}$)



$$\vec{k} = k\sin\theta\cos\phi\hat{x} + k\sin\theta\sin\phi\hat{y} + k\cos\theta\hat{z}$$

The new wavefunctions are now linear combinations of new orbital functions $|X'\rangle, |Y'\rangle, |Z'\rangle$. They can be transormed back to the orbital functions in the fixed coordinate through the following Coordination Transformation:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Example: Heavy Hole Wavefunction

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$u'_{hh} = \left|\frac{3}{2},\frac{3}{2}\right\rangle' = \frac{-1}{\sqrt{2}}\left|(X'+iY')\uparrow'\right\rangle = \frac{-1}{\sqrt{2}}\left|(X'+iY')\right\rangle\left|\uparrow'\right\rangle$$

$$= \frac{-1}{\sqrt{2}}\left|(\cos\theta\cos\phi - i\sin\phi)X + (\cos\theta\sin\phi + i\cos\phi)Y - (\sin\theta)Z\right\rangle\left|\uparrow'\right\rangle$$

$$\bar{u}'_{hh} = \left|\frac{3}{2},\frac{-3}{2}\right\rangle' = \frac{-1}{\sqrt{2}}\left|(X'-iY')\uparrow'\right\rangle = \frac{-1}{\sqrt{2}}\left|(X'+iY')\right\rangle\left|\uparrow'\right\rangle$$

$$= \frac{1}{\sqrt{2}}\left|(\cos\theta\cos\phi + i\sin\phi)X + (\cos\theta\sin\phi - i\cos\phi)Y - (\sin\theta)Z\right\rangle\left|\uparrow'\right\rangle$$

C-HH Matrix Element

Optical Matrix Element:

$$H'_{ba} = -\frac{eA_0}{2m_0}\hat{e}\cdot\langle b|\vec{P}|a\rangle = -\frac{eA_0}{2m_0}\hat{e}\cdot\vec{p}_{cv}\cdot I_{hm}^{en}$$

There are 4 possible terms for C-HH transistion: $\overrightarrow{M}_{C-HH} = \vec{p}_{C-HH}$

$$\langle u'_c|\vec{P}|u'_{hh}\rangle = \frac{-1}{\sqrt{2}}\left[(\cos\theta\cos\phi - i\sin\phi)P_x\hat{x} + (\cos\theta\sin\phi + i\cos\phi)P_y\hat{y} - (\sin\theta)P_z\hat{z}\right]$$

$$= \frac{-P_x}{\sqrt{2}}\left[(\cos\theta\cos\phi - i\sin\phi)\hat{x} + (\cos\theta\sin\phi + i\cos\phi)\hat{y} - (\sin\theta)\hat{z}\right]$$

$$\langle u'_c|\vec{P}|\bar{u}'_{hh}\rangle = 0 \quad \because \quad \langle\uparrow'|\downarrow'\rangle = 0$$

$$\langle\bar{u}'_c|\vec{P}|u'_{hh}\rangle = 0$$

$$\langle\bar{u}'_c|\vec{P}|\bar{u}'_{hh}\rangle = \frac{P_x}{\sqrt{2}}\left[(\cos\theta\cos\phi + i\sin\phi)\hat{x} + (\cos\theta\sin\phi - i\cos\phi)\hat{y} - (\sin\theta)\hat{z}\right]$$

C-HH Matrix Element

To find polarziation Dependence $\Rightarrow$ Integrate over all possible θ and ϕ:

$$\left|\hat{x}\cdot\vec{p}_{C-HH}\right|^2 = \frac{1}{4\pi}\int_0^{\pi}\sin\theta\cdot d\theta\int_0^{2\pi}d\phi\cdot$$

$$\frac{1}{2}\left\{\frac{P_x^2}{2}\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)+\frac{P_x^2}{2}\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)\right\}$$

$$\left|\hat{x}\cdot\vec{p}_{C-HH}\right|^2 = \frac{1}{4\pi}\int_0^{\pi}\sin\theta\cdot d\theta\int_0^{2\pi}d\phi\cdot\left\{\frac{P_x^2}{2}\left(\cos^2\theta\cos^2\phi+\sin^2\phi\right)\right\} = \frac{P_x^2}{3} \equiv M_b^2$$

Similarly,

$$\left|\hat{y}\cdot\vec{p}_{C-HH}\right|^2 = M_b^2$$

$$\left|\hat{z}\cdot\vec{p}_{C-HH}\right|^2 = M_b^2$$

C-HH transistion in Bulk Semiconductor is Polarization Independent!

C-LH Matrix Element

Similarly:

$$\left|\hat{x} \cdot \vec{p}_{C-LH}\right|^2 = M_b^2$$

$$\left|\hat{y} \cdot \vec{p}_{C-LH}\right|^2 = M_b^2$$

$$\left|\hat{z} \cdot \vec{p}_{C-LH}\right|^2 = M_b^2$$

C-LH transistion in Bulk Semiconductor is Polarization Independent!

$\Rightarrow$

Bulk Semiconductor is Polarization Independent!

Values of Matrix Element

To find polarziation Dependence $\Rightarrow$ Integrate over all possible θ and ϕ:

$$M_b^2 = \frac{P_x^2}{3} = \frac{1}{3}\frac{m_0^2}{\hbar^2}P^2 = \frac{1}{3}\frac{m_0^2}{\hbar^2}\left(1-\frac{m_e^*}{m_0}\right)\left(\frac{\hbar^2 E_g(E_g+\Delta)}{2m_e^*(E_g+\frac{2}{3}\Delta)}\right)$$

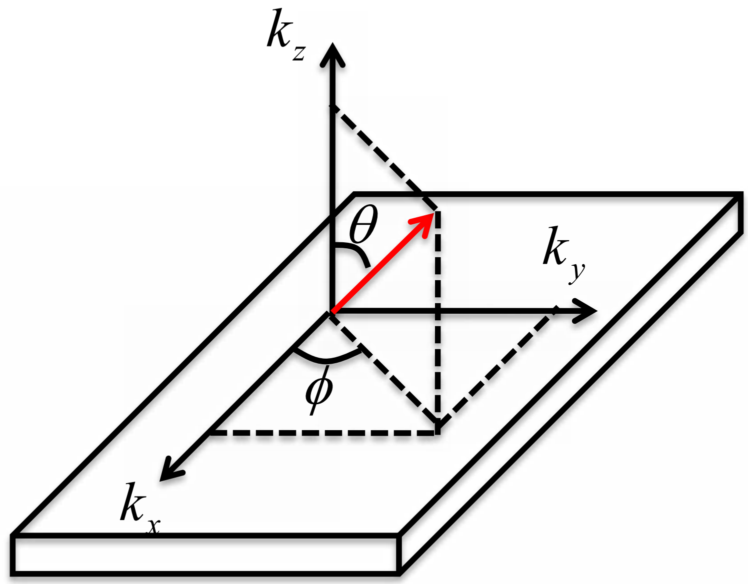
$$\left(1-\frac{m_e^*}{m_0}\right) \approx 1$$

$$M_b^2 \approx \frac{m_0^2 E_g(E_g+\Delta)}{6m_e^*(E_g+\frac{2}{3}\Delta)} \propto \frac{E_g}{m_e^*}$$

$$M_b^2 \approx \frac{m_0}{6}\left(\frac{m_0}{m_e^*}\frac{E_g(E_g+\Delta)}{(E_g+\frac{2}{3}\Delta)}\right) \rightarrow \frac{m_0}{6}E_P$$

	E_p
GaAs	25.7 eV
InP	20.7 eV
InAs	22.2 eV

C-HH Matrix Element in Quantum Well



Quantum well is polarization dependent !

Integrate over ϕ only

$$\left|\hat{x}\cdot\vec{p}_{C-HH}\right|^2 = \frac{1}{2\pi}\int_0^{2\pi} d\phi \cdot \left\{\frac{P_x^2}{2}\left(\cos^2\theta\cos^2\phi + \sin^2\phi\right)\right\}$$

$$= \frac{3}{4}\left(1+\cos^2\theta\right)M_b^2$$

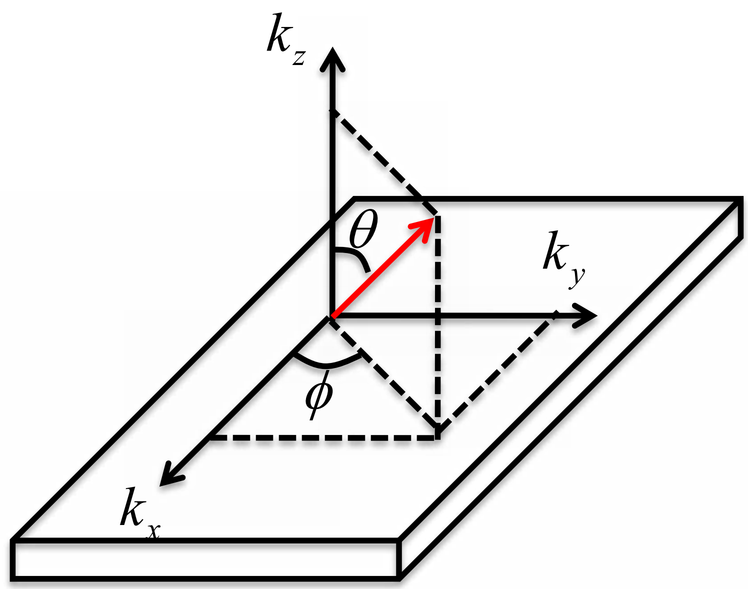
$$\left|\hat{y}\cdot\vec{p}_{C-HH}\right|^2 = \frac{1}{2\pi}\int_0^{2\pi} d\phi \cdot \left\{\frac{P_x^2}{2}\left(\cos^2\theta\sin^2\phi + \cos^2\phi\right)\right\}$$

$$= \frac{3}{4}\left(1+\cos^2\theta\right)M_b^2$$

$$\left|\hat{z}\cdot\vec{p}_{C-HH}\right|^2 = \frac{1}{2\pi}\int_0^{2\pi} d\phi \cdot \left\{\frac{P_x^2}{2}\left(\sin^2\theta\right)\right\}$$

$$= \frac{3}{2}\left(\sin^2\theta\right)M_b^2$$

C-LH Matrix Element in Quantum Well



Quantum well is polarization dependent !

Integrate over ϕ only :

$$\left|\hat{x}\cdot\vec{p}_{C-LH}\right|^2 = \left(\frac{5}{4}-\frac{3}{4}\cos^2\theta\right)M_b^2$$

$$\left|\hat{y}\cdot\vec{p}_{C-HH}\right|^2 = \left(\frac{5}{4}-\frac{3}{4}\cos^2\theta\right)M_b^2$$

$$\left|\hat{z}\cdot\vec{p}_{C-HH}\right|^2 = \left(\frac{1}{2}+\frac{3}{2}\cos^2\theta\right)M_b^2$$